Gaussian fluctuations in the two-dimensional BCS-BEC crossover

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Outline

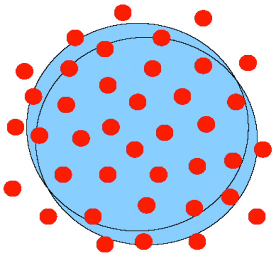
- Introduction and motivation: BCS-BEC crossover in 2D.
- Theoretical description of a 2D Fermi gas: mean-field and Gaussian fluctuations.
- The role of fluctuations: the composite boson limit.
- Results and comparison with experimental data:
    - First sound
    - Second sound
    - Berezinskii-Kosterlitz-Thouless critical temperature.

Main reference: GB and L. Salasnich, arXiv:1507.07542.

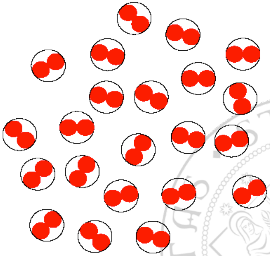
The BCS-BEC crossover (1/2)

In 2004 the **BCS-BEC crossover** has been observed with ultracold gases made of fermionic ^{40}K and ^{6}Li alkali-metal atoms. The fermion-fermion attractive interaction can be tuned (using a Feshbach resonance), from weakly to strongly interacting.

BCS regime: weakly interacting Cooper pairs.

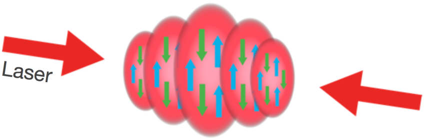


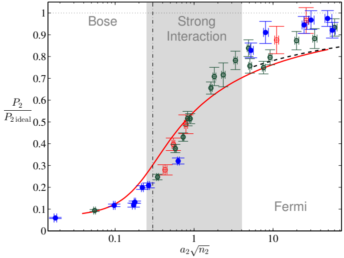
BEC regime: tightly bound bosonic molecules.



The BCS-BEC crossover (2/2)

An additional laser confinement can used to create a quasi-2D pancake geometry. The 2D scattering length is determined by the geometry[1]:
$a_{2D} \simeq \ell_z \exp(-\sqrt{\pi/2}\ell_z/a_{3D}) \geq 0$





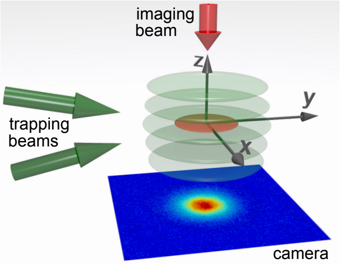
In 2014 the 2D BCS-BEC crossover has been achieved[1] with a **quasi-2D Fermi gas of ^{6}Li atoms** with widely tunable s-wave interaction. The pressure P *vs* the gas parameter $a_B n^{1/2}$ has been measured.

[1] V. Makhalov, K. Martiyanov, and A. Turlapov, PRL **112**, 045301 (2014).
[2] ℓ_z is the thickness of each layer.

The BCS-BEC crossover in 2D (1/2)

Many properties of 2D Fermi gases are currently being studied:

- **Imaging of the atomic cloud[1].**
- Phase diagram[1].
- Very recently (June 2015) the direct observation of the BKT transition has been reported[2].
- Dynamical properties: sound velocity.



[1] M. G. Ries et al., Phys. Rev. Lett. **114**, 230401 (2015)

[2] P. A. Murthy et al., Phys. Rev. Lett. **115**, 010401 (2015).

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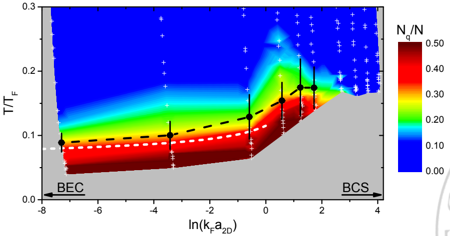

[1] M. G. Ries et al., Phys. Rev. Lett. **114**, 230401 (2015)

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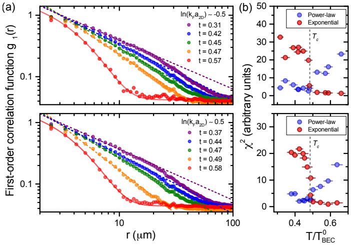

[1] M. G. Ries et al., Phys. Rev. Lett. **114**, 230401 (2015)

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The BCS-BEC crossover in 2D (2/2)

Why is the 2D case interesting from the theory point of view?

- The fluctuations are more relevant for lower dimensionalities. The mean field theory can correctly describe (to some extent) the crossover in 3D, we expect it not to work at all in 2D.
- Berezinskii-Kosterlitz-Thouless mechanism:
  - Mermin-Wagner-Hohenberg theorem: no condensation at finite temperature, no off-diagonal long-range order.
  - Algebraic decay of correlation functions $\langle \exp(i\theta(\mathbf{r})) \exp(i\theta(0)) \rangle \sim |\mathbf{r}|^{-\eta}$
  - Transition to the normal state at a finite temperature T_{BKT}.
- The physics of the BCS-BEC crossover is also relevant in the description of many different systems (bilayers of dipolar gases, exciton condensates). It may also be relevant for the description of high-T_c cuprates as the scaled correlation length ($k_F\xi_0 \sim 5$ for YBCO and $k_F\xi_0 \sim 10$ for LSCO) lies between the BCS ($k_F\xi_0 \sim 10^3$) and BEC ($k_F\xi_0 \ll 1$) regimes.

Formalism for a D-dimensional Fermi superfluid (1/4)

We adopt the path integral formalism. The partition function $\mathcal{Z}$ of the uniform system with fermionic fields $\psi_s(\mathbf{r},\tau)$ at temperature T, in a D-dimensional volume L^D, and with chemical potential μ reads

$$\mathcal{Z} = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \; \exp\left\{ -\frac{1}{\hbar} \; S \right\} ,$$

where ($\beta \equiv 1/(k_B T)$ with k_B Boltzmann's constant)

$$S = \int_0^{\hbar\beta} d\tau \int_{L^D} d^D\mathbf{r} \; \mathcal{L}$$

is the Euclidean action functional with Lagrangian density:

$$\mathcal{L} = \bar{\psi}_s \left[\hbar\partial_\tau - \frac{\hbar^2}{2m}\nabla^2 - \mu \right] \psi_s + g_0 \, \bar{\psi}_\uparrow \, \bar{\psi}_\downarrow \, \psi_\downarrow \, \psi_\uparrow$$

where g_0 is the attractive strength ($g_0 < 0$) of the s-wave coupling.

Formalism for a D-dimensional Fermi superfluid (2/4)

In 2D the strength of the attractive s-wave potential is $g_0 < 0$, which can be implicitely related to the bound state energy:

$$-\frac{1}{g_0} = \frac{1}{2L^2} \sum_{\mathbf{k}} \frac{1}{\epsilon_k + \frac{1}{2}\epsilon_b} \, .$$

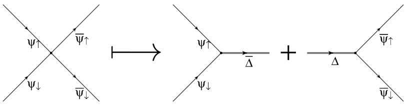
with $\epsilon_k = \hbar^2 k^2/(2m)$. In 2D, as opposed to the 3D case, a bound state exists even for arbitrarily weak interactions, making ϵ_B a good variable to describe the whole BCS-BEC crossover.

The binding energy ϵ_b and the fermionic (2D) scattering length a_{2D} are related by the equation[2]:

$$\epsilon_B = \frac{4\hbar^2}{e^{2\gamma} m a_{2D}^2}$$

[2] C. Mora and Y. Castin, Phys. Rev. A **67**, 053615 (2003).

Formalism for a D-dimensional Fermi superfluid (3/4)



Through the usual Hubbard-Stratonovich transformation the Lagrangian density $\mathcal{L}$, quartic in the fermionic fields, can be rewritten as a quadratic form by introducing the auxiliary complex scalar field $\Delta(\mathbf{r},\tau)$ so that:

$$\mathcal{Z} = \int \mathcal{D}[\psi_s,\bar{\psi}_s]\,\mathcal{D}[\Delta,\bar{\Delta}]\ \exp\left\{-\frac{S_e(\psi_s,\bar{\psi}_s,\Delta,\bar{\Delta})}{\hbar}\right\},$$

where

$$S_e(\psi_s,\bar{\psi}_s,\Delta,\bar{\Delta}) = \int_0^{\hbar\beta} d\tau \int_{L^D} d^D\mathbf{r}\ \mathcal{L}_e(\psi_s,\bar{\psi}_s,\Delta,\bar{\Delta})$$

and the (exact) effective Euclidean Lagrangian density $\mathcal{L}_e(\psi_s,\bar{\psi}_s,\Delta,\bar{\Delta})$ reads

$$\mathcal{L}_e = \bar{\psi}_s\left[\hbar\partial_\tau - \frac{\hbar^2}{2m}\nabla^2 - \mu\right]\psi_s + \bar{\Delta}\,\psi_\downarrow\,\psi_\uparrow + \Delta\bar{\psi}_\uparrow\,\bar{\psi}_\downarrow - \frac{|\Delta|^2}{g_0}\,.$$

Formalism for a D-dimensional Fermi superfluid (4/4)

We want to investigate the effect of fluctuations of the pairing field $\Delta(\mathbf{r}, t)$ around its saddle-point value Δ_0 which may be taken to be real. For this reason we set

$$\Delta(\mathbf{r}, \tau) = \Delta_0 + \eta(\mathbf{r}, \tau) \; ,$$

where $\eta(\mathbf{r}, \tau)$ is the complex field which describes pairing fluctuations. In particular, we are interested in the grand potential Ω, given by

$$\Omega = -\frac{1}{\beta} \ln\left(\mathcal{Z}\right) \simeq -\frac{1}{\beta} \ln\left(\mathcal{Z}_{mf} \mathcal{Z}_g\right) = \Omega_{mf} + \Omega_g \; ,$$

where

$$\mathcal{Z}_{mf} = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \, \exp\left\{ -\frac{S_e(\psi_s, \bar{\psi}_s, \Delta_0)}{\hbar} \right\}$$

is the mean-field partition function and

$$\mathcal{Z}_g = \int \mathcal{D}[\psi_s, \bar{\psi}_s] \, \mathcal{D}[\eta, \bar{\eta}] \; \exp\left\{ -\frac{S_g(\psi_s, \bar{\psi}_s, \eta, \bar{\eta}, \Delta_0)}{\hbar} \right\}$$

is the partition function of Gaussian pairing fluctuations.

Single particle and collective excitations

One finds that in the gas of paired fermions there are two kinds of elementary excitations: fermionic single-particle excitations with energy

$$E_{sp}(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + \Delta_0^2} \, ,$$

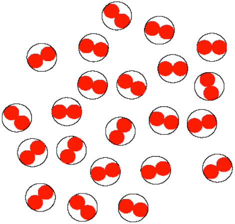
where Δ_0 is the pairing gap, and bosonic collective excitations with energy

$$E_{col}(q) = \sqrt{\frac{\hbar^2 q^2}{2m}\left(\lambda \, \frac{\hbar^2 q^2}{2m} + 2mc_s^2\right)} \, ,$$

where λ is the first correction to the familiar low-momentum phonon dispersion $E_{col}(q) \simeq c_s \hbar q$ and c_s is the sound velocity.

The role of Gaussian fluctuations and collective excitations: composite bosons

In the strongly interacting limit an attractive Fermi gas maps to a gas of composite bosons with chemical potential $\mu_B = 2(\mu + \epsilon_b/2)$ and mass $m_B = 2m$. Residual interaction.
Is this limit correctly recovered[3] at mean-field?
And at a Gaussian level?



Gaussian fluctuations are crucial in correctly describing the properties of a 2D Fermi gas in the BEC limit (boson-boson scattering length, equation of state). What can be said about the sound velocity and the BKT critical temperature?

[1] L. Salasnich and F. Toigo, Phys. Rev. A **91**, 011604(R) (2015)

Regularization

The contribution from fluctuations does not converge:

$$\Omega_g = \frac{1}{2} \sum_{\mathbf{q}} E_{col}(q)$$

Many regularization schemes:

- Dimensional regularization
  Analytical results[4] *in the BEC limit in 2D*
- Counterterms regularization
  Analytical results[5] *in the BEC limit in 3D*
- Convergence factor regularization
  Numerics for the whole crossover[6,7]

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[4] L. Salasnich and F. Toigo, Phys. Rev. A **91**, 011604(R) (2015).
[5] L. Salasnich and GB, Phys. Rev. A **91**, 033610 (2015).
[6] R. B. Diener, R. Sensarma, and M. Randeria, Phys. Rev. A **77**, 023626 (2008)
[7] L. He, H. Lü, G. Cao, H. Hu and X.-J. Liu, arXiv:1506.07156

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First sound velocity (1/2)

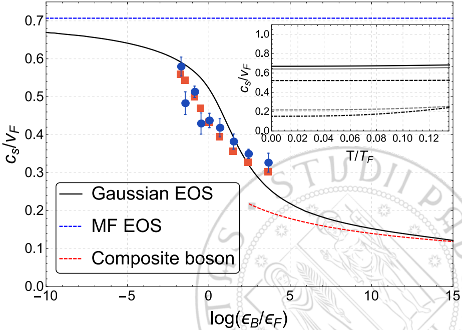
It can be read from the collective excitations spectrum:

$$E_{col}(q) = \sqrt{\frac{\hbar^2 q^2}{2m}\left(\lambda\,\frac{\hbar^2 q^2}{2m} + 2m{c_s}^2\right)} \simeq c_s \hbar q$$

The sound velocity at $T = 0$ can be calculated through the thermodynamics formula:

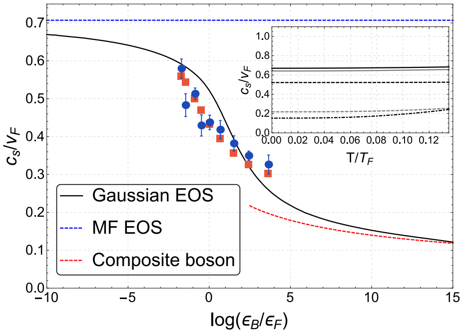
$$c_s = \sqrt{\frac{n}{m}\frac{\partial \mu}{\partial n}}$$

We compare our result with the *"mean-field"* result, with the composite boson limit and with experimental data[1].



[1] N. Luick, M.Sc. thesis, University of Hamburg (2014).

First sound velocity (2/2)



- In the BEC limit c_s is strongly affected by the Gaussian equation of state.
- The temperature dependence (inset) is very weak.
- Strong coupling: composite boson limit.

$$c_s^2 = \frac{8\pi\hbar^2}{m_B} \frac{m_B}{\ln\left(\frac{1}{n_B a_B^2}\right)}$$

- Quite good agreement with (preliminary) experimental data.

BKT critical temperature (1/3)

The BKT critical temperature is found using the Kosterlitz-Nelson (KN) condition:

$$k_B T_{BKT} = \frac{\hbar^2 \pi}{8m} n_s(T_{BKT})$$

The superfluid density is obtained using Landau's quasiparticle excitations formula for fermionic and bosonic excitations:

$$n_{n,f} = \beta \int \frac{\mathrm{d}^2 k}{(2\pi)^2} k^2 \frac{e^{\beta E_k}}{(e^{\beta E_k} + 1)^2} \quad \text{and} \quad n_{n,b} = \frac{\beta}{2} \int \frac{\mathrm{d}^2 q}{(2\pi)^2} q^2 \frac{e^{\beta \omega_q}}{(e^{\beta \omega_q} - 1)^2},$$

then $n_s = n - n_{n,f} - n_{n,b}$.

BKT critical temperature (2/3)

- **Approximation:** the single-particle and collective contributions are not independent, as there is hybridization due to Landau damping. Strictly speaking the bosonic contribution to n_n should be[8]:

$$n_{n,b} = -\frac{m}{\beta}\sum_q \frac{1}{\left(\det\tilde{\mathbb{M}}\right)^2}\left[\det\tilde{\mathbb{M}}\left(\frac{\partial^2 \det\tilde{\mathbb{M}}}{\partial Q^2}\right)_{\tilde{\mu}} - \left(\frac{\partial \det\tilde{\mathbb{M}}}{\partial Q}\right)^2_{\tilde{\mu}}\right]_{Q\to 0}$$

  It reduces to the simpler form seen before in the low-temperature limit, being most relevant at $k_B T \sim \epsilon_F$. In 2D below T_{BKT} $k_B T \lesssim 0.125\epsilon_F$ and the hybridization can be safely ignored.

- **Composite boson limit:** Combining $a_B = \frac{1}{2^{1/2}e^{1/4}}a_F$, $\epsilon_B = \frac{4}{e^{2\gamma}}\frac{\hbar^2}{m{a_F}^2}$ we get:

$$\frac{\epsilon_B}{\epsilon_F} = \frac{\kappa}{n_B a_B^2} \qquad \kappa \simeq 0.061$$

The strongly interacting Fermi gas maps to a dilute Bose gas of dimers.

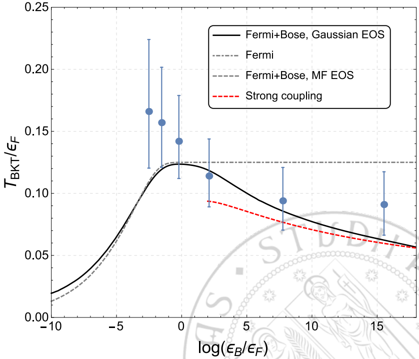
[8] E. Taylor, A. Griffin, N. Fukushima, Y. Ohashi, Phys. Rev. A **74**, 063626 (2006)

BKT critical temperature (3/3)

We can compare the theory with very recently obtained experimental data[9]:

- Within error bars for $\epsilon_B/\epsilon_F \gtrsim 1$
- Worse agreement for $\epsilon_B/\epsilon_F \lesssim 1$
- In the strong coupling limit the KN condition leads to:

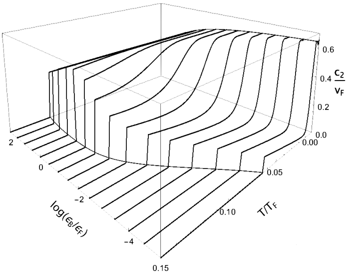
$$k_B T_{BKT} \approx \frac{\mu_B^{\frac{2}{3}} \epsilon_F^{\frac{1}{3}}}{\sqrt[3]{12\zeta(3)}} - \frac{8}{3} \frac{\mu_B^{\frac{4}{3}} \epsilon_F^{-\frac{1}{3}}}{(12\zeta(3))^{\frac{2}{3}}}$$



Caveat: non-2D geometry of the trap.

[1] P. A. Murthy et al., Phys. Rev. Lett. **115**, 010401 (2015).

Second sound velocity



A superfluid can also sustain the second sound (entropy wave as opposed to density wave). Using the same approximation as before, we model the free energy as:

$$F_{sp} = -\frac{2}{\beta} \sum_{\mathbf{k}} \ln \left[1 + e^{-\beta E_{sp}(k)} \right]$$

$$F_{col} = \frac{1}{\beta} \sum_{\mathbf{q}} \ln \left[1 - e^{-\beta E_{col}(q)} \right]$$

The second sound velocity is readily calculated from the entropy as:

$$S = -(\partial F / \partial T)_{N,L^2} \qquad c_2 = \sqrt{\frac{1}{m} \frac{\bar{S}^2}{\left(\frac{\partial \bar{S}}{\partial T}\right)_{N,L^2}} \frac{n_s}{n_n}}$$

Conclusions

- The theoretical treatment of a 2D Fermi gas needs the inclusion of Gaussian fluctuations, which in turn require a proper regularization.
- This approach shows good agreement with experimental data (BKT critical temperature, first sound), other predictions are open to verification (second sound): two-dimensional BCS-BEC is a young field.
- This treatment can be extended to 2D systems with BCS-like pairing (bilayers of polar molecules, exciton condensates, etc.)

Thanks for your attention.