Giacomo Bighin

Università degli Studi di Padova

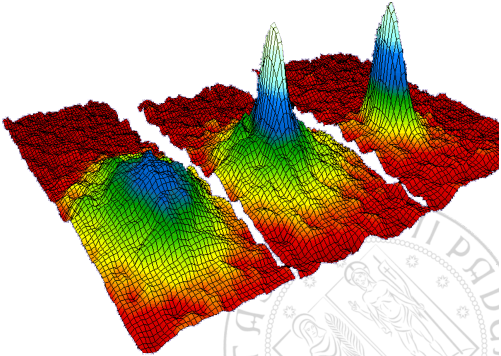
June 5, 2014

Plan of the talk

- Condensate fraction for a polarized Fermi gas.
- A gauge approach to superconductivity in high-T_c cuprates.

Ultracold Fermi gases (1/3)

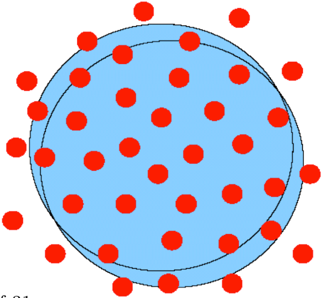
- Ultracold gases: experimental observation of quantum properties of matter. Vortices in a superfluid, BEC.
- Bose-Einstein condensation (1995), degenerate Fermi gas and fermionic condensate (2003).
- Very clean experimental environment: control over the temperature, the number of particles, the interaction.



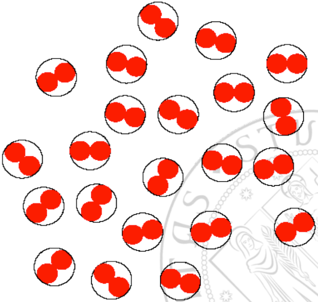
Ultracold Fermi gases (2/3)

Why are ultracold Fermi gases interesting? The fermion-fermion interaction can be tuned (using a Feshbach resonance), from weakly to strongly interacting: the **BCS-BEC crossover**.

BCS regime: coherence in momentum space.

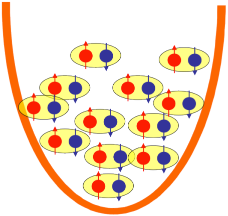


BEC regime: coherence in coordinate space.

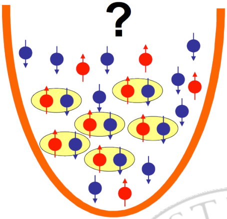


Ultracold Fermi gases (3/3)

Balanced Fermi gas



Polarized Fermi gas



$$\begin{aligned}\mathcal{L} &= \sum_{\sigma=\uparrow,\downarrow} \bar{\psi}_\sigma(\mathbf{r},\tau)\left(\hbar\frac{\partial}{\partial\tau} - \hbar^2\frac{\nabla^2}{2m} - \mu_{\sigma}\right)\psi_\sigma(\mathbf{r},\tau) + \\ &+ g\,\bar{\psi}_\uparrow(\mathbf{r},\tau)\,\bar{\psi}_\downarrow(\mathbf{r},\tau)\,\psi_\downarrow(\mathbf{r},\tau)\,\psi_\uparrow(\mathbf{r},\tau)\end{aligned}$$

Condensate fraction (1/2)

- Why? The condensate fraction is *the* fundamental signature of Bose-Einstein condensation: a finite fraction of particles occupying the ground state.
- Definitions:
  - In terms of the Green functions in Nambu-Gor'kov space:

$$N_0 = \frac{1}{\beta^2} \sum_{\mathbf{p}} \sum_{n} \sum_{m} \mathbb{G}_{21}\left(\mathbf{p}, \mathrm{i}\omega_n\right) \mathbb{G}_{12}\left(\mathbf{p}, \mathrm{i}\omega_m\right)$$

  - In terms of the BCS variational parameters, for $T = 0$:

$$N_0 = 2 \sum_{\mathbf{k}} u_{\mathbf{k}}^2 v_{\mathbf{k}}^2 = \sum_{\mathbf{k}} \frac{\Delta_0^2}{4E_{\mathbf{k}}^2} \qquad E_{\mathbf{k}} = \sqrt{\left(\hbar^2 \frac{k^2}{2m} - \mu\right)^2 + \Delta_0^2}$$

The condensed fraction in the balanced case

Usual path-integral treatment:

$$\mathcal{L} = \sum_{\sigma=\uparrow,\downarrow} \bar{\psi}_\sigma(\mathbf{r},\tau)\left(\hbar\frac{\partial}{\partial\tau} - \hbar^2\frac{\nabla^2}{2m} - \mu\right)\psi_\sigma(\mathbf{r},\tau) + g\bar{\psi}_\uparrow(\mathbf{r},\tau)\bar{\psi}_\downarrow(\mathbf{r},\tau)\psi_\downarrow(\mathbf{r},\tau)\psi_\uparrow(\mathbf{r},\tau)$$

$$\mathcal{Z} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S[\psi,\bar{\psi}]} \qquad S[\psi,\bar{\psi}] = \int_0^\beta \mathrm{d}\tau \int_V \mathrm{d}^3\mathbf{r}\mathcal{L}$$

- Hubbard-Stratonovich transformation: $\Delta(\mathbf{r},\tau) \sim \bar{\psi}\psi$
- Mean field approximation: $\Delta(\mathbf{r},\tau) = \Delta_0 + \cancel{\delta(\mathbf{r},\tau)}$

$$E_\mathbf{k} = \sqrt{\xi_\mathbf{k}^2 + |\Delta_0|^2}$$

Gap equation/number equation from the thermodynamic potential:

$$\frac{\partial\Omega}{\partial\Delta_0} = 0 \qquad n = -\left.\frac{\partial\Omega}{\partial\mu}\right|_{T,\zeta}$$

The condensate fraction in the balanced case: theory vs. experiments

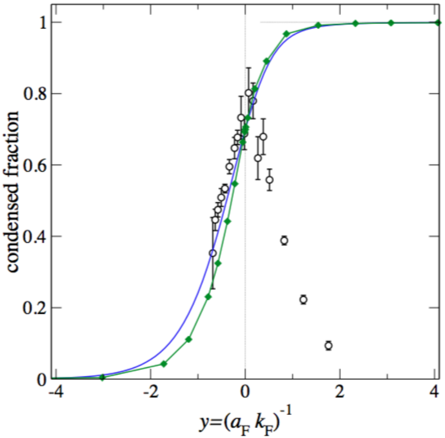


Figure : Condensate fraction $\frac{N_0}{N/2}$ of Fermi pairs in the uniform two-component dilute Fermi gas as a function of $y = (k_F a_s)^{-1}$ (solid line), T=0. The same quantity computed in the LDA for a droplet of $N = 6 \times 10^6$ fermions in harmonic trap, as in the MIT experiment, plotted against the value of y at the center of the trap (joined diamonds). Open circles with error bars: experimentally determined condensed fraction by MIT group. [L. Salasnich, N. Manini, A. Parola, PRA **72**, 023621 (2005)]

The condensate fraction in the balanced case: theory vs. experiments

The comparison between mean-field theory and MIT experiments for the condensate fraction of unpolarized two-component Fermi superfluid suggests that:

- There are no relevant differences between uniform and trapped theoretical results.
- The experimental data are in good agreement with the theory only in the BCS side of the crossover up to unitarity.
- The experimental data on the condensate fraction are not reliable in the BEC regime (inelastic losses?).

Condensate fraction for a unbalanced Fermi gas: theory

Two number equations:

$$n = -\left.\frac{\partial\Omega}{\partial\mu}\right|_{T,\zeta} \qquad \delta n = -\left.\frac{\partial\Omega}{\partial\zeta}\right|_{T,\mu}$$

$$\left(\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2}, \zeta = \frac{\mu_\uparrow - \mu_\downarrow}{2}, \Delta_0\right) \leftrightarrow \left(N, P = \frac{N_+ - N_-}{N_+ + N_-}, y = (k_F a_s)^{-1}\right)$$

Condensate fraction (T=0):

$$N_0 = \sum_{|\mathbf{k}|\notin[k_-,k_+]} \frac{\Delta_0^2}{4E_\mathbf{k}^2} \qquad E_\mathbf{k}^\pm = \sqrt{\xi_\mathbf{k}^2 + |\Delta_0|^2} \pm \zeta$$

Condensate fraction for a unbalanced Fermi gas: theory

Phase separation: it is essential to model the trapping.
In order to compare our theoretical condensate fraction with the MIT experiment done with trapped ^{6}Li atoms, we use the local density approximation (LDA), given by:

$$\mu \to \mu - V(\mathbf{r})$$

where $V(\mathbf{r}) = \frac{m}{2}\left(\omega_\perp^2\left(x^2+y^2\right)+\omega_z^2 z^2\right)$ is the external trapping potential. In this way the gap $\Delta(\mathbf{r})$, the total density $n(\mathbf{r})$ and the condensate density $n0(\mathbf{r})$ become local scalar fields.

Condensate fraction for a unbalanced Fermi gas: theory

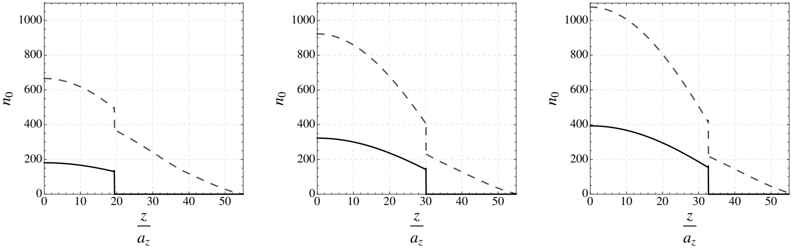


Figure : Condensate density profile $n_0(z)$ (solid line) and total density profile $n(z)$ (dashed line) in the axial direction z for three different scattering lengths. From left to right: $y = -0.44$, $y = 0.0$, $y = 0.11$, where $y = (k_F a_s)^{-1}$ with $k_F = (3\pi^2 n(\mathbf{0}))^{\frac{1}{3}}$ and $n(\mathbf{0})$ the total density at the center of the trap. Number of atoms $N = 2.3 \times 10^7$ and polarization $P = (N_\uparrow - N_\downarrow)/N = 0.2$. Here $a_z = \frac{1}{\sqrt{m\omega_z}}$ is the characteristic length of the axial harmonic confinement.

Condensate fraction for a unbalanced Fermi gas: theory vs. experiments

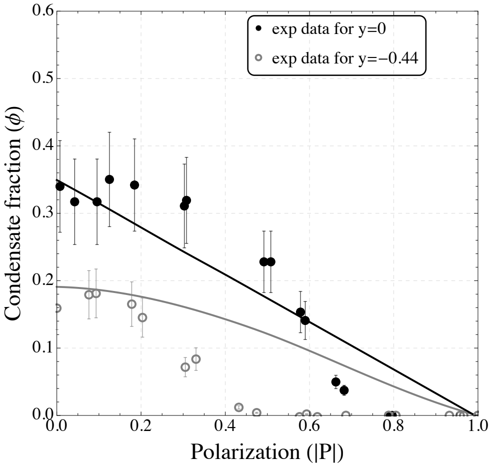


Figure : Condensate fraction ϕ as a function of the absolute value of the polarization $|P|$ for two values of the dimensionless interaction parameter $y = (k_F a_s)^{-1}$: $y = -0.44$ (open circles) and $y = 0.0$ (filled circles), T=0. Circles with error bars are experimental data of ^6Li atoms taken from MIT experiment. Solid lines are our theoretical calculations for the trapped system.

Condensate fraction in the unbalanced case

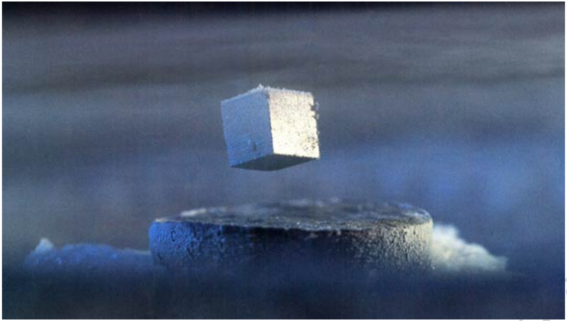
- The polarized case is different! A mean-field theory here generally just gives qualitative agreement.
- The condensate fraction as a function of the polarization agrees with experimental data only for low polarization values.
- We overestimate the phase boundary (determined by the free energy) and, as a consequence, we also overestimate the condensate fraction of a trapped system.

Further work is needed!

Clearly a mean-field description can be satisfying in the balanced case, as far as the condensate fraction is concerned, but it is not enough to correctly reproduce experimental results in the unbalanced case. Current work is on:

- Including the Gaussian fluctuations in the condensate fraction (with Giovanni Lombardi).
- Are the fluctuations enough to reproduce experimental data?
- By doing this we can get more insight on the unbalanced Fermi gas: why is it different? Why do we need to include the fluctuations?

High-T_c superconductivity in cuprates



Main reference: P. A. Marchetti, F. Ye, Z. B. Su, and L. Yu Phys. Rev. B **84**, 214525

Cuprates: an overview (1/2)

- **Superconducting cuprates**: a class of superconducting materials with very high critical temperatures (up to 135 °K).
- Discovered in 1986 by J. G. Bednorz e K. A. Müller; Nobel prize awarded in 1987, the fastest in history.
- Very active research field: more than 100,000 research articles in ~ 25 years.
- Up to date, the microscopical mechanism behind SC in cuprates is not completely understood.

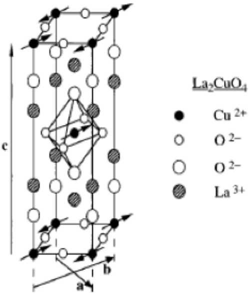
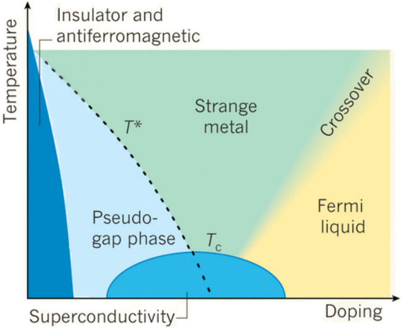


Figure : Unitary cell for La_2CuO_4.

Cuprates: an overview (2/2)

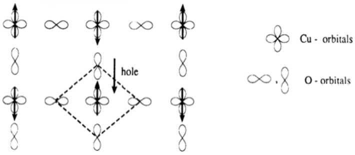
- Different chemical compositions (YBCO, LSCO, BSSCO) the only common chemical features being the CuO_2 planes. $\Longrightarrow$ The CuO_2 planes are believed to be the main seat of superconductivity.



- Dependence on doping and universality for the **phase diagram**.
- BCS theory can not account for SC in cuprates.

From the CuO_2 planes to the t/J model

CuO_2 planes in terms of Zhang-Rice singlets:



ZR: Doping-induced hole reside (primarily) on combinations of four oxygen p orbitals centered around a copper site.

From ZR singlets to the t/J model:

- Strong on-site repulsion (P_G)
- Nearest neighbour hopping ($t \approx 0.3\,\mathrm{eV}$)
- Anti-ferromagnetic Heisenberg term ($J \approx 0.1\,\mathrm{eV}$)

$$H_{\mathrm{t/J}} = \sum_{\langle i,j \rangle} P_G \left[-t \sum_{\alpha} c^{\dagger}_{i\alpha} c_{j\alpha} + h.c. + J \mathbf{S}_i \cdot \mathbf{S}_j \right] P_G$$

"Doping a Mott insulator", P.A. Lee, N. Nagaosa, X.-G. Wen, Rev. Mod. Phys. **78**, 17

Spin-charge separation

The creation/annihilation operators for the electron are rewritten in terms of a product of two operators:

$$\hat{c}_{i,\alpha} = \hat{s}_{i,\alpha}\hat{h}_i^\dagger$$

where

- $\hat{h}_i$ is a spinless fermion (holon): the P_G constraint is always satisfied due to Pauli exclusion principle.
- $\hat{s}_{i,\alpha}$ is a spin $\frac{1}{2}$ boson (spinon).

A new local invariance is introduced by this process:

$$U(1)_{h/s} \quad \begin{cases} \hat{s}_{i,\alpha} \longrightarrow \hat{s}_{i,\alpha} e^{\mathrm{i}\phi(x)} \\ \hat{h}_i \longrightarrow \hat{h}_i e^{\mathrm{i}\phi(x)} \end{cases}$$

Emergent $U(1)$ gauge field: $A_\mu \approx s_\alpha^* \partial_\mu s_\alpha + \cdots$

Chern-Simons bosonization

What is a bosonization scheme?
Simplest example: Jordan-Wigner transformation (1D) on lattice:

$$c_j^\dagger \longrightarrow a_j^\dagger e^{-\mathrm{i}\pi \sum_{l<j} a_l^\dagger a_l}$$

Continuum version (1D):

$$\Psi(x) \longrightarrow \Phi(x)\, e^{\mathrm{i}\int_{\gamma_x} A_\mu(y)\mathrm{d}y^\mu}$$

- **Basic idea** for Chern-Simons bosonization in 2D: just like in the 1D case one can bind to the newly-introduced bosonic operator to a "string" which restores the correct statistics back.
- In 2D a gauge field and additional term in the action are needed.

Effective action

In 2D one can add a charge flux Φ, modifying the statistics:

$$c_{j\alpha} = e^{-\mathrm{i}\Phi_h(j)} h_j^* \left(e^{\mathrm{i}\Phi_s(j)} s_j \right)_\alpha$$

This decomposition is exact; but why do we divide the electron's degrees of freedom in this way?

- Which are the fundamental excitations of t/J model?
- The charge and spin degrees of freedom can be divided in many different ways (e.g. slave boson/slave fermion), however the MF results are very different.
- Hints from the 1D case (Marchetti et al., Nuclear Physics B, **482** (1996), 731-757)
- Energetically this is the best choice.

Result: an effective description of the t/J model in terms of spinons and holons.

Towards superconductivity

The electron has a composite structure:
spinon + holon

$\longmapsto$

Superconductivity is achieved in three stages:

- Holon pairing
- Spinon pairing
- Phase coherence

The pairing process (1/2)

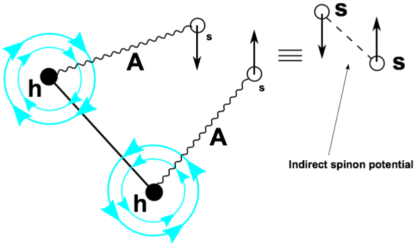


Figure : The attractive potential between the spinons, essential for the SC, is mediated by a gauge field "binding" holon and spinons, and by the holon attraction.

The superconductivity is achieved in **three steps**: holon pairing (T_{ph}), spinon pairing (T_{ps}), phase coherence (T_c):

$$\Delta_c \sim \frac{|\Delta^s|}{|\Delta^h|} e^{\mathrm{i}\overbrace{(\phi_s - \phi_h)}^{\equiv\phi}}$$

$$SC \Longleftrightarrow \langle \Delta_c \rangle \neq 0$$

The pairing process (2/2)

- The spinon-spinon interaction is repulsive, the gauge fluctuations play a key role.
- The condensate phase dynamics is described by a gauged 3DXY (Stueckelberg) model.
- Δ_s is the spinon order parameter. Dispersion relation for spinons: $E_{\pm}(\mathbf{k}) = \sqrt{m_s^2 + |\mathbf{k}|^2 \pm 2|\Delta^s||\mathbf{k}|}$
- The SC transition is essentially XY, corresponding to the transition between the two phases of the 3DXY model (Coulomb/Higgs), with effective temperature: $\Theta \propto |\Delta_0^s|^{-2}$.
- We observe three characteristics temperatures:
  - T_{ph}: holon pairing.
  - T_{ps}: spinon pairing
  - T_c: phase coherence.

Superfluid density

- Superfluid density (ρ_s). $S_{\mathrm{EFF}} = \frac{\rho_s}{2} \int \mathrm{d}\tau \mathrm{d}^d r \, (\nabla\theta)^2 + \cdots$
- Importance:
  - Lots of experimental data, $\rho_s \propto \lambda^{-2}$.
  - Very different from BCS.
  - Empirical relations (Uemura relation: $\rho_s(T=0) \propto T_c$).
- Summation formula ($\sim$ Ioffe-Larkin)

$$\rho_s = \frac{\rho_s^s \rho_s^h}{\rho_s^s + \rho_s^h}$$

- Our results are in very good agreement with experimental data. The critical exponent is exactly reproduced:
  $\rho_{\mathrm{s}} \sim \left| \frac{T-T_c}{T} \right|^{\frac{2}{3}} \quad \text{for } T \longrightarrow T_c$

Comparison with experimental data (1/3)

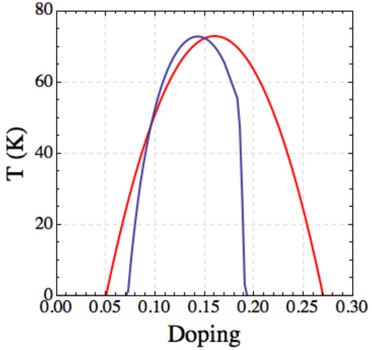


Figure : The superconductive dome, experimental data (red) vs. our model (blue).

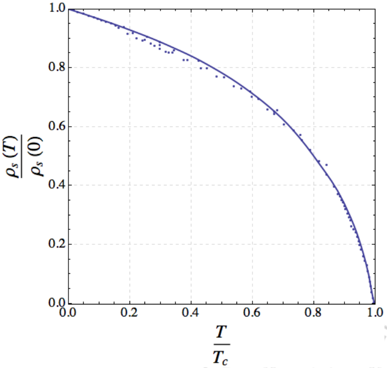


Figure : (Rescaled) superfluid density as a function of the rescaled temperature vs. data from YBCO. The critical exponent is $\frac{2}{3}$.

Comparison with experimental data (2/3)

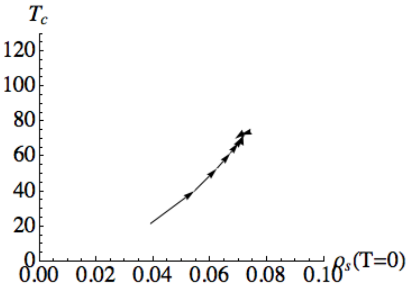


Figure : Uemura observed strong linearity between $\rho_s(T = 0)$ and T_c in underdoped cuprates ($0 \leq \delta \leq 0.15$). Our theoretical model correctly reproduces this feature.

Comparison with experimental data (3/3)

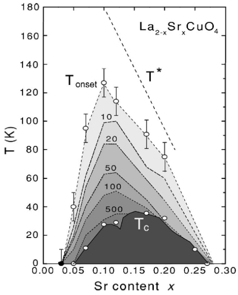


Figure : The phase diagram of LSCO showing the Nernst region between T_c and T_{onset}, in Phys. Rev. B **73**, 024510.

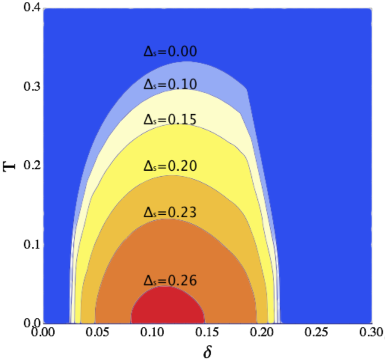


Figure : The Nernst signal can be interpreted in terms of a incoherent condensate. Temperature in units of $J = 1300K$

Conclusion

- Proposed mechanism of pairing-SC: based on composite holon-spinon nature of the hole, realized with a three-step scenario.
- A summation rule for the superfluid density has been derived; the general behavior of ρ_s is in very good agreement with experiments.
- Other experimental features of cuprates are correctly reproduced.

Thanks for your attention.